Middle School Geometry
Session 2

Topic	Activity Name	Page Number	Related SOL	Activity Sheets	Materials
Spatial Relationships	Square It	52	6.14, 7.9	Square It	Playing board, grid paper, colored one-inch square markers
	Pick Up the Toothpicks	54	6.14	Pick Up the Toothpicks	Toothpicks
	Partition the Square	56	6.14, 6.15, 7.9	Partition the Square	Paper
Tangrams	Make Your Own Tangrams	59	6.13, 6.14, 6.15, 7.9, 7.11	Directions for Making Tangrams	Paper, scissors
	Area and Perimeter Problems/Tangrams	61	6.11, 7.7	Area and Perimeter with Tangrams	Tangram set
	Spatial Problem Solving with Tangrams	64	6.14, 6.15, 7.9	Problem Solving with Tangrams Sheets 1, 2	Tangram set, puzzles
Soma Cubes	Constructing the Soma Pieces	68	6.17, 7.8, 8.7, 8.8, 8.9	Instructor Reference Sheet, Soma Views, Soma Pieces: Surface Area and Volume	27 wooden cubes, sugar cubes, or snap cubes/participant, permanent markers
	Building the Soma Cube	73	6.17, 7.8, 8.7, 8.8, 8.9	Build This Cube, Soma Solutions Recording Sheet	7 Soma pieces from previous lesson
	Making 2-D Drawings of 3-D Figures	77	6.17, 8.9	Isometric Dot Paper	7 Soma pieces from previous lesson
	Cube Structures	79	6.17, 8.9		One-inch cubes

Topic: Spatial Relationships

Description: To build spatial visualization skills, students need a wide variety of experiences, including building and dissecting figures from different perspectives. In these activities, participants will explore spatial relationships by playing a strategy game with squares, solving a toothpick triangle puzzle, partitioning squares into smaller squares, and using square dissection puzzles.

Related SOL: 6.14, 6.15, 7.9

Activity: Square It

Format: Small Group

Objectives: Participants will recognize squares and gain practice in visualization. As an extension, students will determine the area of a square by counting the number of square units needed to cover it.

Related SOL: 6.14, 7.9

Materials: Playing board, Square It Activity Sheet or 8 x 11 one-inch grid paper, and colored one-inch square markers of two different colors.

Time Required: Approximately 15 minutes

Directions:

1) Participants play this game in pairs. Players choose who starts.

2) Player places a marker of his or her color on a vacant box on the playing board. Players alternate placing markers.

3) The winner is the player to first recognize a SQUARE on the board where all four corners are his or her color. Players check for "squareness" by counting the lengths of the sides. Winning squares may range from 2 x 2 to 7 x 7.

4) (Optional) The winning player must state the area of the winning square.

Square It

Activity: Pick Up the Toothpicks

Format: Small Group

Objectives: Participants will recognize triangles and gain practice in spatial visualization.

Related SOL: 6.14

Materials: 11 toothpicks per participant, Pick Up The Toothpicks Activity Sheet

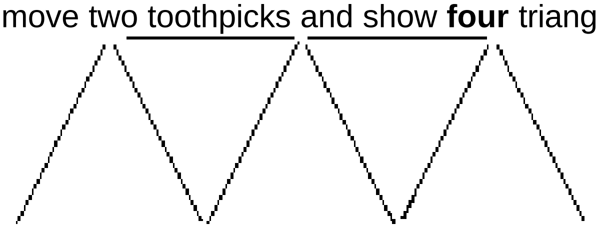
Time Required: Approximately 10 minutes

Directions:

1) Pass out 11 toothpicks per participant. Tell them to use the toothpicks as they work through the problems posed.

2) Discuss the directions on the Activity Sheet. Eleven toothpicks are arranged as shown to give five triangles. For each problem, begin with the original 11-stick configuration. Then:

   A. remove two toothpicks and show **three** triangles;
   B. remove one toothpick and show **four** triangles;
   C. remove three toothpicks and show **three** triangles; and
   D. remove two toothpicks and show **four** triangles.



3) Be sure to discuss the fact that all the sides don't have to be the same length in order for a three-sided figure to be a triangle.

Pick Up the Toothpicks

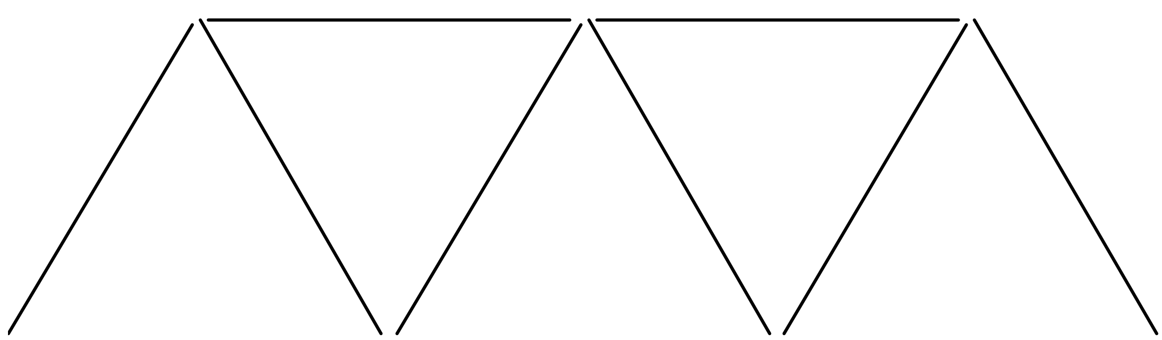
Eleven toothpicks are arranged as shown to give five triangles. For each problem, begin with the original 11-stick configuration. Then:

A. remove two toothpicks and show **three** triangles;

B. remove one toothpick and show **four** triangles;

C. remove three toothpicks and show **three** triangles; and

D. remove two toothpicks and show **four** triangles.



Activity: Partition the Square

Format: Individual /Large Group

Objectives: Participants will partition squares into smaller squares and will explain how they know that the smaller figures are really squares.

Related SOL: 6.14, 6.15, 7.9

Materials: Partition the Square Activity Sheet, scratch paper

Time Required: Approximately 30 minutes

Directions:

1) Distribute the Partition the Square Activity Sheet. Explain to the participants that they are to divide each square into smaller squares and that there are many ways to determine each number of squares. Use the two sample partitions to point out that the squares don't have to be the same size, just that four sides of each square must be congruent, and that overlaps will not count.

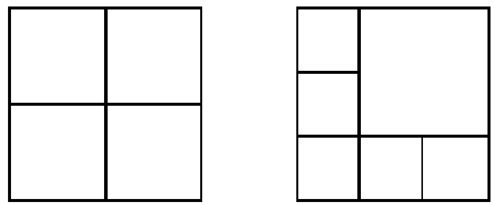
2) Circulate around the room, referring participants to the two samples if they need assistance. Also, look for non-square rectangles and remind the participants that all four sides of a square are congruent.

3) After they have had a few minutes to work, ask the participants to share their solutions. Start out with labels 7 to 15 and ask for a volunteer to do each one. Ask participants to add their method if they have a different way of partitioning than the one shown.

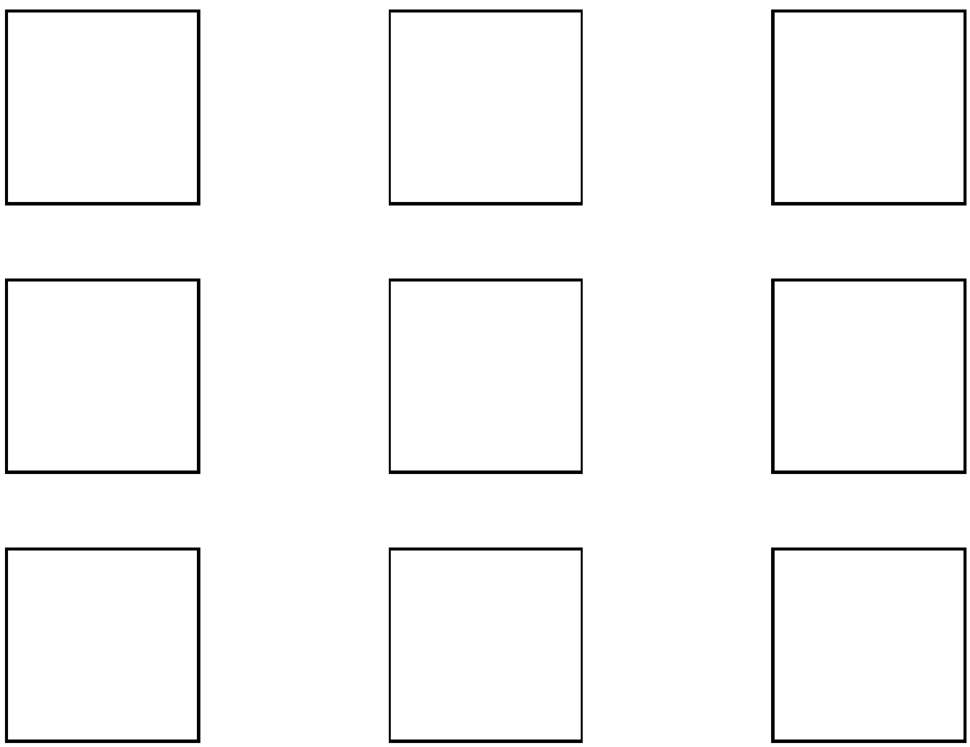
4) After all solutions have been shared by the participants, challenge the group to justify that each really is composed of squares. Tell them that you will allow them to assume that angles that look like right angles are right angles. Perhaps the simplest way of justifying four congruent sides in each of the drawn squares is to think of the original square as a unit square and then label the sides accordingly.

Partition the Square

A square can be partitioned into squares in more than one way. Shown below are squares partitioned into 4 smaller squares and 6 smaller squares.



Use these partitioning ideas to find ways to partition the nine squares below into 7 to 15 smaller squares.



Topic: Tangrams

Description: Participants will make their own tangrams and will use them to explore area and perimeter relationships in geometric figures. They will engage in problem-solving puzzles using tangrams.

Related SOL: 6.11, 6.13, 6.14, 6.15, 7.7, 7.9

Activity: Make Your Own Tangrams

Format: Small Group

Objectives: Participants will construct their own tangrams and identify properties of the seven tangram pieces.

Related SOL: 6.13, 6.14, 6.15, 7.9, 7.11

Materials: Paper suitable for folding such as copy paper (one sheet per student), scissors, one set of overhead tangrams, Directions for Making Tangrams Activity Sheet

Time Required: 30 minutes

Directions:

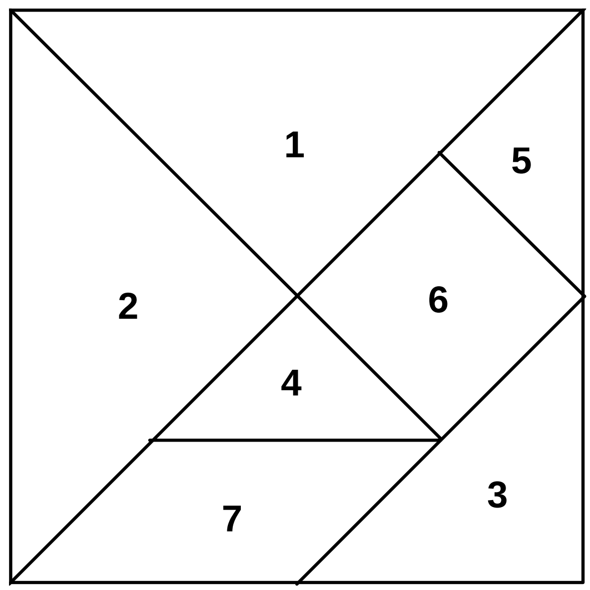
1) Distribute directions for making a set of the seven tangram pieces (Directions for Making Tangrams Activity Sheet) or give the directions orally for participants to follow individually as you make a master set as a demonstration.

2) Ask participants to make a square out of all seven tangram pieces.

3) Have participants label the pieces by number as indicated in the diagram below. They should also identify each by the name of the figure. Put a set of overhead tangrams on the overhead projector and discuss:
   - Identify each tangram piece by the name of the figure.
   - Which figures are congruent? How do you know?
   - Which triangles are similar? How do you know? Can you write a proportion to express the relationship between the lengths of the triangle sides?

4) Have participants find the measure of each angle of each figure. They should trace each figure and label the angle measures.

Directions for Making Tangrams



1) Fold the lower right corner to the upper left corner along the diagonal. Crease sharply. Cut along the diagonal.

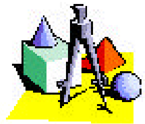
2) Fold the upper triangle formed in half, bisecting the right angle, to form Piece 1 and Piece 2. Crease and cut along this fold. Label these two triangles "1" and "2."

3) Connect the midpoint of the bottom side of the original square to the midpoint of the right side of the original square. Crease sharply along this line and cut. Label the triangle "3."

4) Fold the remaining trapezoid in half, matching the short sides. Cut along this fold.

5) Take the lower trapezoid you just made and connect the midpoint of the longest side to the vertex of the right angle opposite it. Fold and cut along this line. Label the small triangle "4" and the remaining parallelogram "7."

6) Take the upper trapezoid you made in Step 4. Connect the midpoint of the longest side to the vertex of the obtuse angle opposite it. Fold and cut along this line. Label the small triangle "5" and the square "6."



Activity: Area and Perimeter Problems with Tangrams

Format: Individual/Small group

Objectives: Participants will explore area relationships with tangrams.

Related SOL: 6.11, 7.7

Materials: A set of tangrams for each participant, Area and Perimeter with Tangrams Activity Sheet

Time Required: 30 - 40 minutes

Directions:

1) Give participants the Area and Perimeter with Tangrams Activity Sheet. Make sure they also have a set of tangrams. Ask them to work alone or in small groups to complete the tasks outlined on the Activity Sheet. Circulate around the room, helping participants who need assistance.

2) Invite participants to the overhead projector to describe how they found the answers.

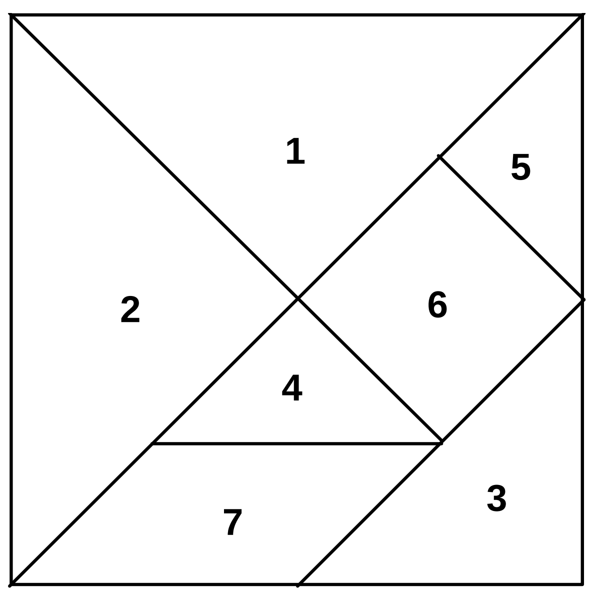
Area and Perimeter With Tangrams

1) If the area of the composite square (all seven pieces -- see below) is one unit, find the area of each of the separate pieces in terms of the area of the composite square.

Piece #	area
1	
2	
3	
4	
5	
6	
7	

2) If the smallest triangle (piece #4 or #5) is the unit for area, find the area of each of the separate pieces in terms of that triangle.

Piece #	area
1	
2	
3	
4	
5	
6	
7	



3) If the smallest square (piece #6) is the unit for area, find the area of each of the separate pieces in terms of that square. Enter your findings in the table below.

4.) If the side of the small square (piece #6) is the unit of length, find the perimeter of each piece and enter your findings in the table.

piece #	area	perimeter
1		
2		
3		
4		
5		
6		
7		

Activity: Spatial Problem Solving with Tangrams

Format: Independent/Small Group

Objectives: Participants will create geometric figures with the tangram pieces.

Related SOL: 6.14, 6.15, 7.9

Materials: A set of tangrams for each student; Spatial Problem Solving with Tangrams Activity Sheet, Tangram Puzzles Activity Sheet

Time Required: Variable, allow 30 minutes to get started. Participants may work independently over a period of a week or so and turn in solutions at a later session.

Directions: Distribute Activity Sheets and have participants work individually or in small groups to solve the tangram puzzles.

Spatial Problem Solving with Tangrams

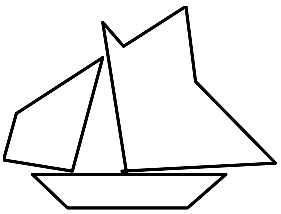
Use the number of pieces in the first column to form each of the geometric figures that appear in the top of the table. Make a sketch of your solution(s). Some have more than one solution while some have no solution.

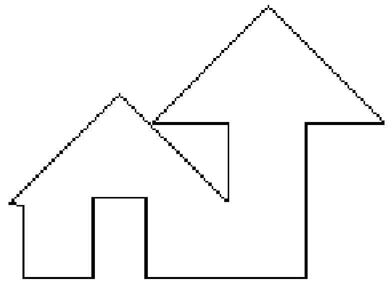
Make These Polygons

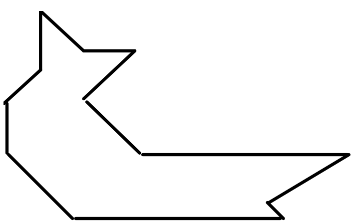
Use this many pieces	Square	Rectangle	Triangle	Trapezoid	Trapezoid	Parallel -ogram
2						
3						
4						
5						
6						
7						

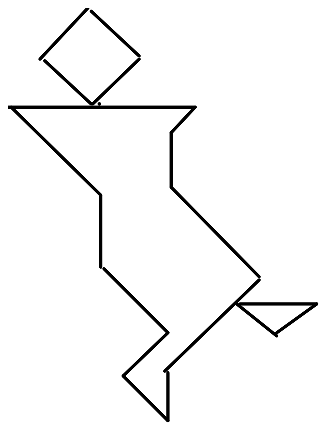
Tangram Puzzles

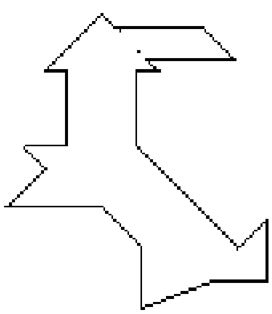
Can you make these figures using all seven tangram pieces? Make a sketch of your solutions.

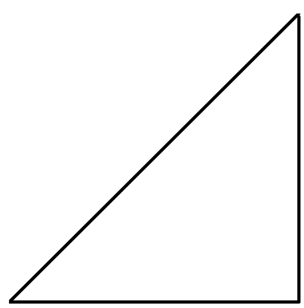
1) 

2) 

3) 

4) 

5) 

6) 

Design your own tangram picture. Trace the outline and give it a name. Submit the outline and a solution key.

Topic: Spatial Relations Using the Soma Cube

Description: Participants will construct the seven Soma pieces and solve problems involving the pieces.

Related SOL: 6.17, 7.8, 8.7, 8.8, 8.9

Activity: Constructing the Soma Pieces

Format: Whole group directions and discussion followed by individual problem-solving.

Objectives: Participants will solve spatial problems and will find surface area and volume of constructed figures.

Related SOL: 6.17, 7.8, 8.7, 8.8, 8.9

Materials: 27 wooden cubes for each student (or sugar cubes, or Snapcubes), glue, permanent markers; glue, and markers; Instructor Reference Sheet; Soma Views from Top, Front, and Side Activity Sheet; Surface Area and Volume of the Soma Pieces Activity Sheet

Time Required: 45 minutes

Directions:

1) Give out 27 wooden 1-inch cubes or 27 snapcubes to each participant or small group of participants if materials are limited. Present them with the following problem:

    - How many different ways can you join three cubes face-to-face? These are called "tricubes."

    Have participants try finding them with the cubes and discuss the results. Tell them that if a tricube can be flipped or repositioned (reflected, rotated) in such a way that it is exactly like a tricube already made, then it is not different from the other one.

    There are only two different tricubes. However, for this activity we only need to save the non-rectangular one. Have participants put aside the rectangular one (every face is a rectangle!).

2) Have participants find all possible non-rectangular tetracubes (4 unit cubes joined face-to-face). Discuss what they find. There should be six different ones (see reference page that accompanies this lesson). You may have participants glue the wooden cubes together (or snap the snapcubes together) and number the completed pieces according to the reference page. Discuss the nature of the pieces:

    - Are any of the pieces reflections of each other? (If you put a mirror next to one piece, will you see the other in the mirror?)

- Which pieces can be placed so that they are only one unit high? (Pieces #1, #2, #3, #4)

- Which pieces must occupy space that is 2 units high? (Pieces #5, #6, #7)

- Which of the pieces have a line of symmetry on a given face?

3) Review the concept of volume by identifying one of the wooden cubes or one of the snapcubes as the unit. Discuss the face of the unit cube as the unit of area. Have participants find the surface area and volume of each of the numbered solids #1 - #7 and complete the table in Handout 2.7 Surface Area and Volume Activity Sheet. Discuss their findings:

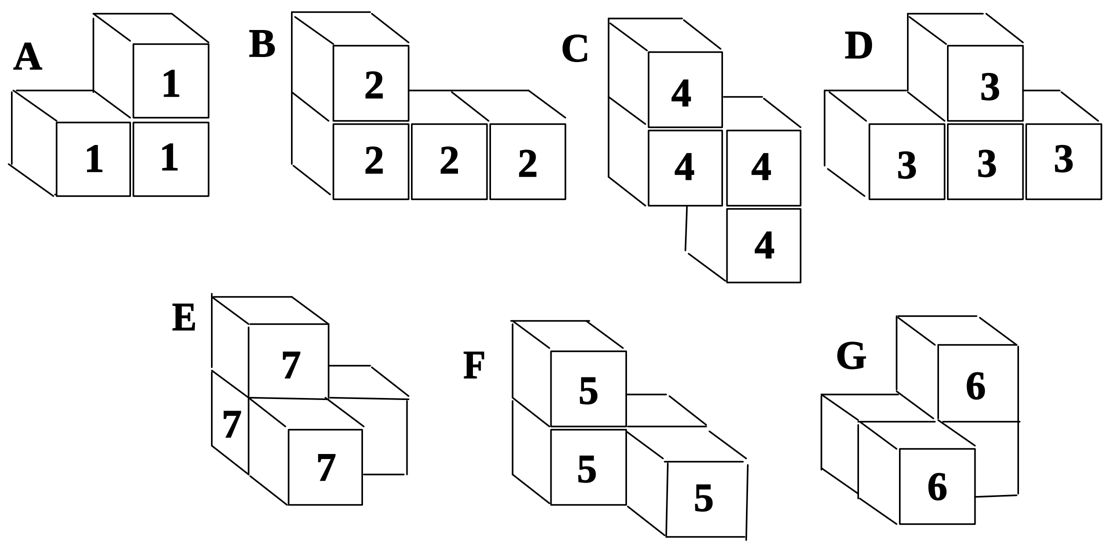
   - Did the pieces with the same volume have the same surface area?

   - Did the pieces with the same surface area have the same volume?

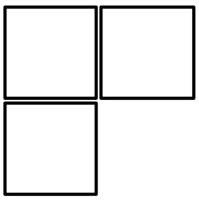
4) Give participants diagrams of the top, side, and bottom view of Soma pieces in Soma Views from the Top, Front, and Side Activity Sheet and have them identify the pieces from these views. Have them draw the diagrams for the remaining pieces.

Extension: Ask participants to build all "pentacubes" that can be made with five cubes each.

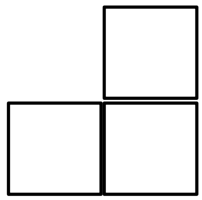
Instructor Reference Sheet



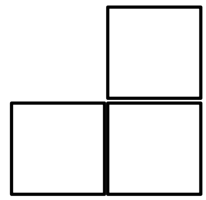
Soma Views from Top, Front, and Side



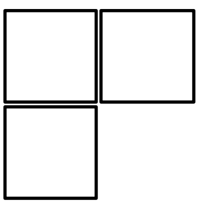
top



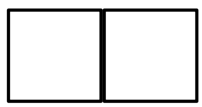
front



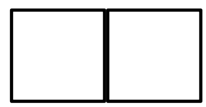
right side



top



front



right side

Surface Area and Volume of the Soma Pieces

piece #	surface area	volume
1		
2		
3		
4		
5		
6		
7		

Activity: Building the Soma Cube and other Structures with Soma Pieces

Format: Small group

Objectives: Participants will use the seven Soma pieces to build the 3x3x3 cube and other structures that use all seven pieces.

Related SOL: 6.17, 7.8, 8.7, 8.8, 8.9

Materials: Seven Soma pieces from previous lesson, Build This Cube and Soma Solutions Recording Sheet Activity Sheets

Time Required: 45 minutes; some participants may want to extend the activities independently

Directions:

1) Give participants some history of the Soma cube. It was invented by Piet Hein in Denmark. He was listening to a lecture on quantum physics when the speaker talked about slicing up space into cubes. Hein then thought about all the irregular shapes that could be formed by combining no more than four cubes, all the same size and joined at their faces. In his head he figured out what these would be and that it would take 27 cubes to build them all. From there he showed that the pieces could form a 3x3x3 cube.

2) Tell participants: *So now we know that the seven pieces fit together to form a 3x3x3 cube. In fact, there are 1,105,920 different ways to assemble the cube. Try to find one.*

   Let participants work until they get a solution.

3) Ask participants how they might make a record of their solution before they take the cube apart. You may show them one way by sharing the solution-recording sheet that accompanies this lesson. It requires that the numbers of the unit cubes be recorded in three layers: top, middle, and bottom. Have participants record their solutions. Note that some will need help in making the correct correspondence of the numbers to the grid. Then have them try to find a different solution and record it.

4) Participants may want to explore the Soma Cube further by going to the web site http://web.inter.nl.net/users/C.Eggermont/Puzzels/Soma/

Discuss:

- How many dimensions are represented in the Soma cube?
- How many dimensions are represented in the solution sheet?
- Does anyone want to share any strategies that might help in transferring the 3-dimensional information onto the 2-dimensional representation?

4) Now reverse the order of the task. Give participants a written Soma solution and ask them to build that cube. The Build This Cube Activity Sheet contains this example:

Top

6	6	2
5	6	2
5	5	2

Middle

6	3	4
5	4	4
7	4	2

Bottom

3	3	3
7	1	1
7	7	1

5) Participants use the Activity Sheet with pictures of structures that can be built with the seven Soma pieces. Once they have succeeded at building any of the structures, they should label the drawings with the numbers of the appropriate pieces.

6) Have participants complete a table in which they compare the volume of the structures and the surface area. Discuss their observations with the whole class:
   - What is the volume of the completed Soma cube?
   - What is the volume of the other structures you built?
   - How can you tell the volume of the structures by only studying the picture and not actually building them?
   - How can you figure out the surface area of the structures without actually building the figures?

Build This Cube

Top

6	6	2
5	6	2
5	5	2

Middle

6	3	4
5	4	4
7	4	2

Bottom

3	3	3
7	1	1
7	7	1

Soma Solutions Recording Sheet

Top Middle Bottom

Top Middle Bottom

Top Middle Bottom

Activity: Making 2-Dimensional Drawings of 3-Dimensional Figures

Format: Participants will work in small groups to apply the drawing techniques that have been demonstrated by the instructor.

Objectives: Participants will use isometric dot paper to make drawings of Soma pieces to make a 2-dimensional drawing of a 3-dimensional figure.

Related SOL: 6.17, 8.9

Materials: Isometric Dot Paper and Soma pieces from previous lesson

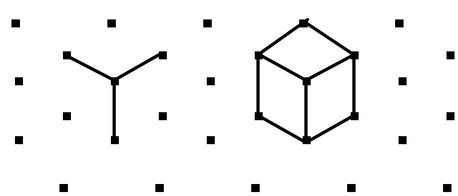
Time Required: 45 minutes

Directions:

1) Distribute the isometric dot paper to participants. Have them study it and discuss how it is different from regular graph paper. Discuss:

   - What does the prefix "iso" mean?
   - How does this apply to the way the paper is designed?

2) Have participants practice drawing single cubes while working in small groups so they can help each other. (Hint: the easiest way to show this is by drawing a Y in the center and then circumscribing a hexagon around it-- below.)

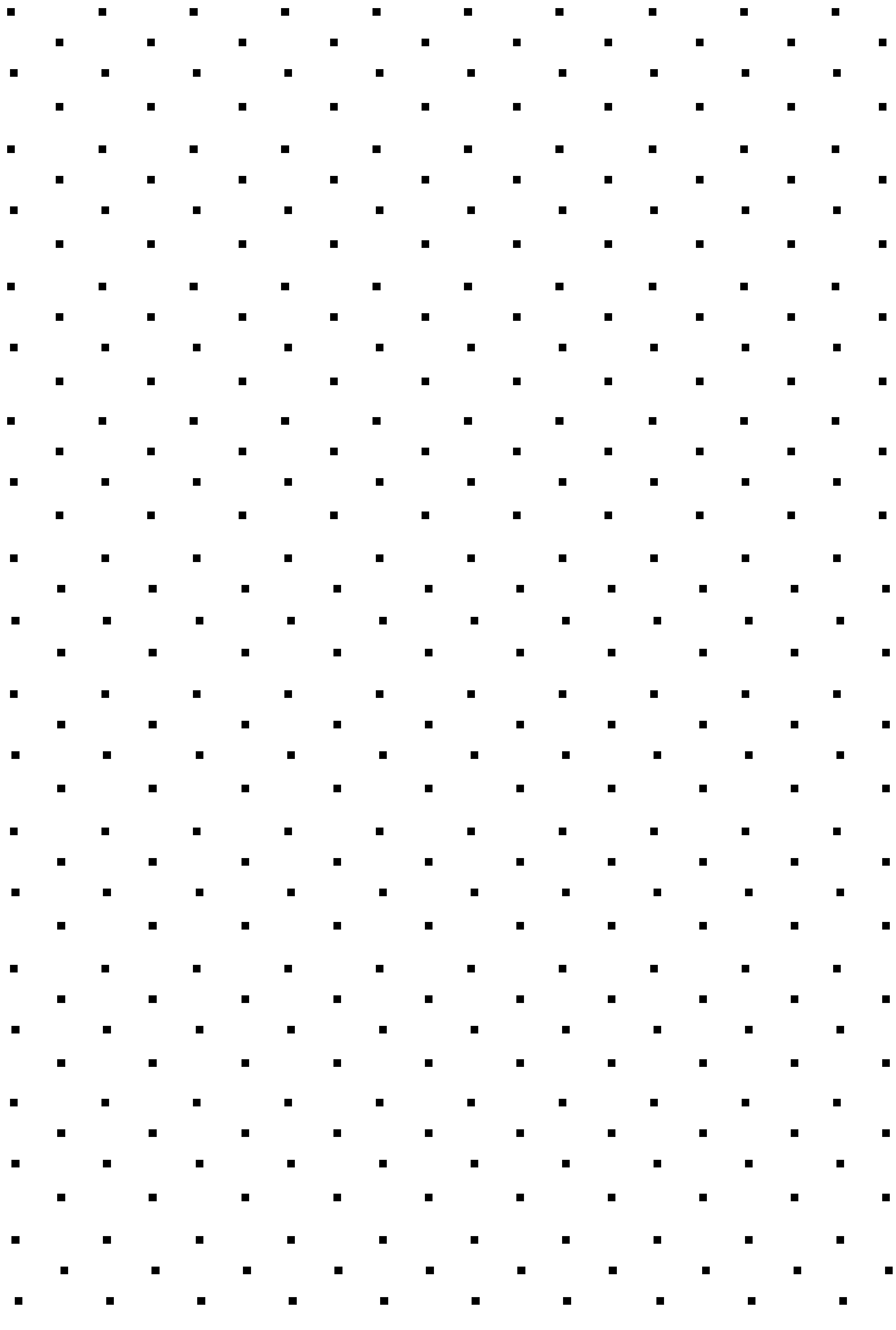


3) Show participants how to position one of the seven Soma pieces on a diagonal so that they can draw it on the isometric paper. Draw one on the overhead while talking through the process.

4) Have participants work in groups to draw all seven Soma pieces on isometric dot paper. Each participant should complete his or her own drawings with the help of group members.

5) Challenge participants to design a structure using all seven Soma pieces and draw it on the isometric dot paper.

Isometric Dot Paper



Activity: Cube Structures

Format: Small group

Objectives: Participants will draw the top, front, and side views of cube structures and will build structures from drawings of the three views.

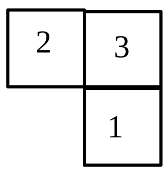
Related SOL: 6.17, 8.9

Materials: One-inch cubes and paper

Time Required: 20 minutes

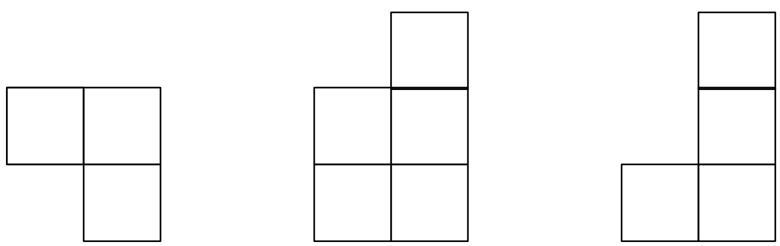
Directions:

1) Give each participant 10 one-inch cubes. Ask them to build a structure that stacks six cubes as indicated below (this is the view from the top).



2) Ask participants to draw the top view, the front view, and the right-side view of the structure. Discuss and check for accuracy.

Key:



Top View Front View Side View

3) Have participants work in pairs with a barrier to block the view of the partner's structure. Each person should build a structure with some of the blocks and then draw the three views -- top, front, and side.

4) Each person should then pass the drawings to the partner. The partners build the structures according to the pictures. Remove the barrier to check the accuracy of the structure.

5) Repeat the procedure with a new structure.